Magnetoexciton dispersion in polar semiconductors

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A study of the exciton binding energy in three and two spatial dimensions in polar crystals in the presence of a strong uniform magnetic field is presented. The calculations are performed within the lowest Landau-level approximation, assuming the existence of Fröhlich interaction between the electrons and the longitudinal optical phonons. It is shown that the magnetoexciton binding energy in a pure two-dimensional polar crystal has the same form as in the nonpolar semiconductors but with an effective dielectric constant which depends on the strength of the magnetic field.

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I. INTRODUCTION

In polar semiconductors the interaction between a single electron and longitudinal optical (LO) phonons dominates as compared with the interaction with phonons of other types. The corresponding interaction is known as the Fröhlich interaction, and it leads to a dielectric constant which depends on the frequency,

$$\varepsilon^{-1}(\omega)=\frac{1}{\varepsilon_\infty}-\frac{\omega_0}{2\varepsilon^*}\left[\frac{1}{\omega_0-\omega-\imath 0^+}+\frac{1}{\omega_0+\omega-\imath 0^+}\right],$$

where ω_0 is the LO optical phonon frequency and $\varepsilon^{*-1}=\varepsilon_\infty^{-1}-\varepsilon_0^{-1}$. In the limit of zero frequency, $\varepsilon(\omega)$ is equal to the static dielectric constant ε_0. For frequencies well above the optical phonon frequency, the dielectric constant approaches the high-frequency dielectric constant ε_∞. Turning our attention to the exciton-LO-phonon system in polar crystals, we find that the basic assumption is that the electron and hole that constitute the exciton are considered well separated, and therefore interact individually with phonons. In other words, the excitons in polar crystals can be described by an extension of the Fröhlich Hamiltonian initially derived for describing the interaction between electrons and phonons. This point of view leads to the conclusion that the exciton dispersion in polar crystals should be calculated using a Schrödinger type of equation for the relative electron-hole motion but with a more complicated interaction between them. The electron-hole interaction in polar crystals has been investigated decades ago by applying different techniques, such as a perturbation theory,[1] unitary transformations,[2] path integrals,[3] a variational method,[4] the Green's function method,[5] the equation-of-motion method,[6] and the dielectric-function method.[7] Each of the abovementioned theoretical methods provides its own potential for the electron-hole interaction, usually a nonlocal one, but regardless of the applied techniques the potential always depends on the static and high-frequency dielectric constants ε_0 and ε_∞, the LO phonon energy $\hbar\omega_0$, and the electron m_c and hole m_v bare masses. Since the experimentally obtained m_c and m_v should be interpreted as polaron-mass parameters, an additional procedure is needed to convert the polaron-mass parameters into the bare-mass parameters.

Strong magnetic fields can dramatically change the properties of excitons. The strong magnetic limit means that the electrons and holes are confined primarily to the lowest Landau level (LLL), and the typical Coulomb energy (the three-dimensional exciton Rydberg) is much smaller than the exciton cyclotron energy. The Hamiltonian that usually is used to calculate magnetoexciton dispersion has the following form:[8,9]

$$\hat{H}_{\rm exc}=E_g+\frac{\hbar^2Q_z^2}{2M}-\frac{\hbar^2}{2\mu}\frac{d^2}{dz^2}-\frac{\hbar^2}{2\mu}\nabla_{\mathbf{r}}^2+\frac{\imath e\gamma\hbar}{2\mu c}(\mathbf{B}\times\mathbf{r})\cdot\nabla_{\mathbf{r}}+\frac{e^2B^2}{8\mu c^2}\mathbf{r}^2+V_C(\mathbf{r}+l^2\mathbf{Q}_0;z),\quad(1)$$

where $l=(\hbar c/eB)^{1/2}$ is the magnetic length. Hamiltonian (1) is written using the center of mass $\mathbf{R}=\alpha_c\mathbf{r}_c+\alpha_v\mathbf{r}_v$ and the relative $\mathbf{r}=\mathbf{r}_c-\mathbf{r}_v$ coordinates. The coefficients $\alpha_c=(1-\gamma)/2$ and $\alpha_v=(1+\gamma)/2$ are expressed in terms of the parameter $\gamma=(m_v-m_c)/(m_c+m_v)$ which accounts for the difference between the electron m_c and the hole m_v masses. The in-plane exciton pseudomomentum is $\hbar\mathbf{Q}=\hbar(Q_x,Q_y,0)$, while $\mathbf{Q}_0=(-Q_y,Q_x,0)$. $M=m_c+m_v$ and $\mu=m_cm_v/(m_c+m_v)$ are the exciton total mass and its in-plane reduced mass, respectively. The electron and the hole dispersion laws are $E_c(\mathbf{k})=E_g+\hbar^2\mathbf{k}^2/2m_c$ and $E_v(\mathbf{k})=\hbar^2\mathbf{k}^2/2m_v$, respectively, where E_g is the semiconductor band gap. Since the above Hamiltonian does not include the interaction with LO phonons, the Coulomb attractive interaction between the electron and the hole, $V_C(\mathbf{r})=-e^2/\varepsilon r$, must be screened by the static dielectric constant. It is possible to justify the use of the Coulomb interaction screened by the static dielectric constant in nonpolar materials, but an effective dielectric constant between ε_∞ and ε_0 should be used for interpretation of optical spectra of polar semiconductors in the presence of a constant magnetic field. In GaAs, for example, $\varepsilon_\infty=10.84$ and the static dielectric constant at very low temperatures is $\varepsilon_0=12.74$.[10] In Ref. 11, however, a value of $\epsilon=12.1$ has been used, and this value provides a very good agreement with the measurements of magnetoexciton dispersion in GaAs-coupled quantum wells at $B=4$ T. One may well ask what exact value should be used in polar materials, such as TlCl and TlBr, where the dielectric constants ε_∞ and ε_0 are quite different (see Table I).

In what follows we extend the dielectric-function method to the magnetoexciton-LO-phonon problem. Our approach

TABLE I. Effective dielectric constant $\epsilon(B)$ for various magnetic fields $B=4,6,\ldots,20$ T calculated according to Eq. (25). The GaAs values of ε_0 and ε_∞ are measured at temperature 75.6 K (Ref. 10).

$\hbar\omega_0$ (meV)	ε_0	ε_∞		$\epsilon(4)$	$\epsilon(6)$	$\epsilon(8)$	$\epsilon(10)$	$\epsilon(12)$	$\epsilon(14)$	$\epsilon(416)$	$\epsilon(18)$	$\epsilon(20)$
36.8	12.74	10.89	GaAS	12.25	12.17	12.11	12.06	12.02	11.98	11.95	11.92	11.90
50.0	11.0	9.1	GaP	10.54	10.46	10.40	10.35	10.31	10.27	10.24	10.21	10.19
43.4	8.6	5.2	ZnS	7.14	6.95	6.81	6.70	6.61	6.54	6.48	6.43	6.38
27.2	7.4	3.7	CuCl	4.89	4.70	4.58	4.50	4.33	4.38	4.34	4.31	4.28
20.8	9.6	7.13	CdTe	8.31	8.18	8.08	8.01	7.96	7.91	7.88	7.84	7.82
21.5	37.6	5.1	TlCl	11.24	9.67	8.82	8.28	7.91	7.63	7.42	7.25	7.11
14.3	35.1	5.4	TlBr	9.10	8.22	7.74	7.43	7.22	7.05	6.93	6.82	6.74

justifies the above choice of the dielectric constant in GaAs and provides an analytical expression which allows us to calculate the dielectric constant in polar materials in strong magnetic field regime. The calculations are done assuming the presence of a strong constant magnetic field **B** along the z axis defined by a vector potential $\mathbf{A}(\mathbf{r})=(1/2)\mathbf{B}\times\mathbf{r}$. As in the case of the absence of a magnetic field, we assume that the electron and the hole, constituting the magnetoexciton, interact individually with the LO phonons.

To the best of our knowledge no attention has been paid to the magnetoexciton dispersion in polar crystals perhaps because of the lack of incentive from the experimental side. The magnetoexcitons in a cylindrical disklike semiconductor quantum dot have been discussed in Ref. 12. The main assumptions in this paper are (i) there is no transverse motion of the magnetoexciton as a whole, (ii) the interaction with phonons in quantum dots is the same as with the Fröhlich bulk LO phonons, and (iii) the magnetoexciton wave functions can be written as the tensorial product of the ground-state noninteracting electron and hole wave functions in the presence of a constant magnetic field. The third assumption is questionable because the electron-hole interaction plays an important role in this case, and therefore, we should expect that the exact eigenfunctions differ significantly from the corresponding noninteracting electron and hole wave functions. This means that in order to obtain reasonable numerical results by diagonalization of the Fröhlich exciton-phonon Hamiltonian, one should use a linear combination of noninteracting electron and hole wave functions which includes not only the ground-state wave functions but also the excited-state wave functions as well. Some attention has been focused on the polaronic effect on the shallow donors in the presence of a strong magnetic field.[13] Shallow donors are the analog of the magnetoexcitons with zero in-plane pseudomomentum $\mathbf{Q}=0$. In both papers,[12,13] the assumption of the absence of a transverse motion of the magnetoexciton as a whole simplifies very much the problem, but it neglects the fact that even a small transverse exciton velocity (or small transverse wave vector **Q**) will induce an electric field in the rest frame of the exciton. This electric field will push the electron and the hole apart, so the binding energy must decrease as the transverse velocity increases. In other words, the magnetic field induces a coupling between the center of mass and the relative internal motion. The coupling effect complicates the calculations, so each of the abovementioned techniques needs major modifications in order to be applied to the magnetoexciton-LO-phonon problem.

In what follows we will modify the dielectric-function method which is based on the assumption that the electron-hole attractive interaction in polar crystals is due to the exchange of the longitudinal photons.[7] Since the photons propagate in the crystal, they interact with the polarization created by the lattice vibrations, and therefore, the system under consideration includes electron, photon, and phonon subsystems. In this approach the interactions between the particles are the electron-photon and phonon-photon interactions. The summation of a sequence of diagrams which represents the photon-phonon interaction (see Fig. 1) leads to the following longitudinal part of the photon Green's function $D_\parallel(\mathbf{q},\omega)=(2\pi\hbar c^2/\omega^2)\varepsilon^{-1}(\mathbf{q},\omega)$ where $\varepsilon^{-1}(\mathbf{q},\omega)$ is the inverse dielectric constant. Let $\omega_\lambda(\mathbf{q})$ be the longitudinal normal modes in the crystal, which can be determined by the solutions of the equation $\varepsilon(\mathbf{q},\omega_\lambda)=0$. For photon energies closed to the resonance $\hbar\omega_\lambda(\mathbf{q})$ the longitudinal photon Green's function assumes the form

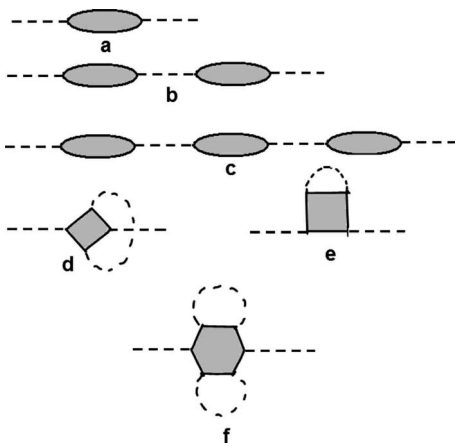

FIG. 1. There are various types of diagrams that represent the interaction of photons with the polarization created by the lattice vibrations. The number of photon lines - - - - is used to classify the diagrams. Parts of diagrams which do not contain photon lines are described by shaded polygons. The contributions of diagrams, such as (d)–(f), are negligibly small (Ref. 14). The sequence of diagrams, such as (a)–(c), has been summed to obtain the longitudinal part of the photon Green's function $D_\parallel(\mathbf{q},\omega)$.

$$D_{\parallel}(\mathbf{q},\omega)=\frac{2\pi\hbar c^2}{\omega_\lambda^2[\partial\varepsilon(\mathbf{q},\omega)/\partial\omega]_{\omega=\omega_\lambda(\mathbf{q})}}\frac{1}{\omega-\omega_\lambda(\mathbf{q})+\imath 0^+}.$$

In the absence of a magnetic field, this approach[7] and the equation-of-motion method[6] both provide similar results for the energy-gap shift, polaron masses, and the exciton binding energy.

Our study of the magnetoexciton-LO-phonon problem is based on the Bethe-Salpeter (BS) equation with a kernel screened by the appropriate dielectric function which takes into account the lattice vibrations. In the presence of a strong magnetic field the LLL approximation for the single-particle Green's function describes qualitatively the main features of the magnetoexcitons in polar crystals. The LLL approximation greatly simplifies the problem because the dynamics of the LLL is essentially $D-2$ dimensional,[15] and only in the LLL approximation we observe a dimensional reduction (from two spacelike coordinates and one timelike coordinate to no spacelike coordinates and one timelike coordinate, i.e., $2+1\rightarrow 0+1$) in the dynamics of fermion pairing in the presence of a constant magnetic field. It is worth mentioning that our approach applied to pure two-dimensional (2D) polar crystals allows us to derive an exact expression for the dielectric constant that could be used to calculate the magnetoexciton binding energy. This is because the dimensional reduction $2+1\rightarrow 0+1$ makes our results independent of the electron and hole masses, and therefore, there is no need of an additional procedure of converting the polaron-mass parameters into the bare-mass parameters. In the absence of a magnetic field this conversion does affect the accuracy of the numerically calculated exciton binding energy because there is no way to measure the electron and hole bare masses in polar materials.

An outline of the paper is as follows. In Sec. II, we study the dispersion of bulk magnetoexcitons. In Sec. III, we consider a quantum-well system assuming the same electron-phonon interaction as in the bulk material.

II. BULK MAGNETOEXCITONS

We shall investigate the role of the interaction with LO phonons by applying the BS formalism widely used in quantum field theory for describing the two-fermion bound states. The basic assumption in the BS formalism is that the electron-hole bound states are described by the BS wave function (BS amplitude) $\Psi(1;2)=\Psi(\mathbf{r}_c,z_c,t_1;\mathbf{r}_v,z_v,t_2)$, where the variables 1 and 2 represent the corresponding coordinates and the time variables. This function determines the probability amplitude to find the electron at the point $(\mathbf{r}_c,z_c)$ at the moment t_1 and the hole at the point $(\mathbf{r}_v,z_v)$ at the moment t_2. The BS amplitude satisfies the following BS equation:

$$\Psi(1;2)=\int d(1',2',1'',2'')G_c(1;1')G_v(2';2)\times I\begin{pmatrix}1' & 1''\\ 2' & 2''\end{pmatrix}\Psi(1'';2'').$$

Here, I is the irreducible BS kernel and $G_{c,v}$ are the electron and the hole single-particle Green's functions. When the screening effects are taken into account, the irreducible kernel represents the screened Coulomb interaction between electrons and holes that constitute the excitons,

$$V(\mathbf{r};z;t)=-\int\frac{d^2\mathbf{q}}{(2\pi)^2}\frac{dq_z}{2\pi}\frac{d\omega}{2\pi}\frac{4\pi e^2}{\sqrt{|\mathbf{q}|^2+q_z^2}}\varepsilon^{-1}(\mathbf{q},q_z,\omega)\times\exp[\imath(\mathbf{q}\cdot\mathbf{r}+q_z z-\omega t)]. \quad (2)$$

In the case when the screening is due to the interaction with bulk LO phonons with frequency ω_0, the inverse dielectric function $\varepsilon^{-1}(\mathbf{q},q_z,\omega)=\varepsilon^{-1}(\omega)$ depends only on the frequency.

The BS equation in the center of mass and reduced coordinates assumes the form

$$\begin{aligned}\Psi_{\mathbf{Q},Q_z}(\mathbf{r},\mathbf{R};z,Z;t,t')&=\int dz'dZ'd^2\mathbf{r}'d^2\mathbf{R}'dt_1dt_2\\&\times G_c\left(\mathbf{R}+\alpha_v\mathbf{r},\mathbf{R}'+\alpha_v\mathbf{r}';Z+\frac{m_{vz}}{M_z}z,Z'+\frac{m_{vz}}{M_z}z';t-t_1\right)\\&\times G_v\left(\mathbf{R}'-\alpha_c\mathbf{r}',\mathbf{R}-\alpha_c\mathbf{r};Z'-\frac{m_{cz}}{M_z}z',Z-\frac{m_{cz}}{M_z}z;t_2-t'\right)\\&\times V(\mathbf{r}';z';t_1-t_2)\Psi_{\mathbf{Q},Q_z}(\mathbf{r}',\mathbf{R}';z',Z';t_1,t_2),\end{aligned}$$

where $G_{c,v}$ are the single-particle Green's functions.

The BS amplitude depends on the relative internal time $t-t'$ and on the "center-of-mass" time,

$$\Psi_{\mathbf{Q},Q_z}(\mathbf{r},\mathbf{R};z,Z;t,t')=\exp\left[-\frac{\imath E(\mathbf{Q},Q_z)}{\hbar}(\alpha_c t+\alpha_v t')\right]\psi_{\mathbf{Q},Q_z}(\mathbf{r},\mathbf{R};z,Z;t-t'), \quad (3)$$

where $E(\mathbf{Q},Q_z)$ is the exciton dispersion. Introducing the time Fourier transforms according to the rule $f(t)=\int_{-\infty}^{\infty}f(\omega)\exp(\imath\omega t)\frac{d\omega}{2\pi}$, we transform the above BS equation into the following form:

$$\begin{aligned}\psi_{\mathbf{Q},Q_z}(\mathbf{r},\mathbf{R};z,,Z;\omega)=\int dz'dZ'd^2\mathbf{r}'d^2\mathbf{R}'\frac{d\Omega}{2\pi}&G_c(\mathbf{R}+\alpha_v\mathbf{r},\mathbf{R}'\\&+\alpha_v\mathbf{r}';Z+\alpha_v z,Z'+\alpha_v z';\hbar\omega\\&+\alpha_c E)G_v(\mathbf{R}'-\alpha_c\mathbf{r}',\mathbf{R}-\alpha_c\mathbf{r};Z'\\&-\alpha_c z',Z-\alpha_c z;\hbar\omega-\alpha_v E)V(\mathbf{r}';z';\omega\\&-\Omega)\psi_{\mathbf{Q},Q_z}(\mathbf{r}',\mathbf{R}';z',Z';\Omega),\end{aligned}$$

where $\psi_{\mathbf{Q},Q_z}(\mathbf{r},\mathbf{R};z,Z;\Omega)$ is the Fourier transform of $\psi_{\mathbf{Q},Q_z}(\mathbf{r},\mathbf{R};z,Z;t)$. Since the translation symmetry is broken by the magnetic field, the single-particle Green's functions can be written as a product of phase factors and translation invariant parts. The phase factor depends on the gauge. In the symmetric gauge we have[16]

$$G_{c,v}(\mathbf{r},\mathbf{r}';z,z';\omega) = e^{\imath(e/\hbar c)\mathbf{r}\cdot\mathbf{A}(\mathbf{r}')}\widetilde{G}_{c,v}(\mathbf{r}-\mathbf{r}';z-z';\omega). \tag{4}$$

The broken translation symmetry requires a phase factor for the BS amplitude,

$$\psi_{\mathbf{Q},Q_z}(\mathbf{r},\mathbf{R};z,Z;\Omega) = e^{\imath(e/\hbar c)\mathbf{r}\cdot\mathbf{A}(\mathbf{R})}\chi_{\mathbf{Q},Q_z}(\mathbf{r},\mathbf{R};z,Z;\Omega). \tag{5}$$

The BS Eq. (3) admits translation invariant solution of the form

$$\chi_{\mathbf{Q},Q_z}(\mathbf{r},\mathbf{R};z,Z;\omega) = e^{-\imath(\mathbf{Q}\cdot\mathbf{R}+Q_zZ)}\widetilde{\chi}_{\mathbf{Q},Q_z}(\mathbf{r};z;\omega). \tag{6}$$

The function $\widetilde{\chi}_{\mathbf{Q},Q_z}(\mathbf{r};z;\omega)$ satisfies the following BS equation:

$$\begin{aligned}\widetilde{\chi}_{\mathbf{Q},Q_z}(\mathbf{r};z;\omega) = \int dz'dZ'd^2\mathbf{r}'d^2\mathbf{R}'\frac{d\Omega}{2\pi}\exp\left\{\frac{\imath e}{\hbar c}[(\mathbf{r}+\mathbf{r}')\cdot A(\mathbf{R}'-\mathbf{R}) + \boldsymbol{\gamma}\mathbf{r}\cdot A(\mathbf{r}')]\right\}\\ \times\widetilde{G}_c[\mathbf{R}-\mathbf{R}'+\alpha_v(\mathbf{r}-\mathbf{r}');Z-Z'+\alpha_v(z-z');\hbar\omega+\alpha_cE]\\ \times\widetilde{G}_v[\mathbf{R}'-\mathbf{R}+\alpha_c(\mathbf{r}-\mathbf{r}');Z'-Z+\alpha_c(z-z');\hbar\omega-\alpha_vE]V(\mathbf{r}';z';\omega-\Omega)\widetilde{\chi}_{\mathbf{Q},Q_z}(\mathbf{r}';z';\Omega).\end{aligned}$$

The substitution $\mathbf{R}'\to\mathbf{R}'+\mathbf{R}+\boldsymbol{\gamma}\mathbf{r}$ provides the following equation for the Fourier transform of the exciton wave function $\widetilde{\chi}_{\mathbf{Q},Q_z}(\mathbf{k};k_z;\omega)=\int dzd^2\mathbf{r}\,\exp[-\imath(\mathbf{k}\cdot\mathbf{r}+k_zz)]\widetilde{\chi}_{\mathbf{Q},Q_z}(\mathbf{r};z;\omega)$:

$$\begin{aligned}\widetilde{\chi}_{\mathbf{Q},Q_z}\left(\mathbf{k}-\frac{\gamma}{2}\mathbf{Q};k_z;\omega\right) = \int\frac{dp_z}{2\pi}\frac{d^2\mathbf{q}}{(2\pi)^2}\frac{d^2\mathbf{p}}{(2\pi)^2}d^2\mathbf{R}\int_{-\infty}^{\infty}\frac{d\Omega}{2\pi}e^{-\imath(\mathbf{q}+\mathbf{Q})\cdot\mathbf{R}}\widetilde{G}_c\left[\frac{1}{2}\mathbf{q}+\mathbf{k}-\frac{e}{\hbar c}A(\mathbf{R});k_z+\alpha_vQ_z;\hbar\omega+\alpha_cE\right]\\ \times\widetilde{G}_v\left[-\frac{1}{2}\mathbf{q}+\mathbf{k}-\frac{e}{\hbar c}A(\mathbf{R});k_z-\alpha_cQ_z;\hbar\omega-\alpha_vE\right]\\ \times V\left\{\mathbf{p}-\left[\mathbf{k}-\frac{2e}{\hbar c}A(\mathbf{R})\right];p_z-k_z;\omega-\Omega\right\}\widetilde{\chi}_{\mathbf{Q},Q_z}\left(\mathbf{p}-\frac{\gamma}{2}\mathbf{Q};p_z;\Omega\right),\end{aligned}$$

where $V(\mathbf{k};k_z;\omega)=-4\pi e^2/(\mathbf{k}^2+k_z^2)\varepsilon^{-1}(\omega)$ and $\widetilde{G}_{c,v}(\mathbf{k};k_z;\hbar\omega)$ are the Fourier transforms of $\widetilde{G}_{c,v}(\mathbf{r};z;\hbar\omega)$.

In the effective-mass approximation the exact fermion Green's functions $G_{c,v}$ are replaced by the corresponding propagators of the free fermions but with renormalized masses. The translation invariant parts $\widetilde{G}_{c,v}$ in the Landau-level representation have the following forms:

$$\widetilde{G}_c(\mathbf{r};z;\hbar\omega) = \int\frac{d^2\mathbf{k}}{(2\pi)^2}\frac{dk_z}{2\pi}\widetilde{G}_{c,v}(\mathbf{k};k_z;\hbar\omega)\exp[\imath(\mathbf{k}\cdot\mathbf{r}+k_zz)],$$

$$\widetilde{G}_c(\mathbf{k};k_z;\hbar\omega) = 2\sum_{n=0}^{\infty}\frac{(-1)^n\exp(-l^2\mathbf{k}^2)L_n(2l^2\mathbf{k}^2)}{\hbar\omega-[\hbar^2k_z^2/2m_c+E_g+\hbar\Omega_c(n+1/2)]+\imath0^+},$$

$$\widetilde{G}_v(\mathbf{k};k_z;\hbar\omega) = \sum_{n=0}^{\infty}\frac{(-1)^n\exp(-l^2\mathbf{k}^2)L_n(2l^2\mathbf{k}^2)}{\hbar\omega+[\hbar^2k_z^2/2m_v+\hbar\Omega_v(n+1/2)]-\imath0^+}.$$

Here, $L_n(x)$ are the Laguerre polynomials and $\hbar\Omega_{c,v}=\hbar eB/cm_{c,v}$ are the electron and hole cyclotron energies. In strong magnetic fields the probability for transitions to the excited Landau levels due to the Coulomb interaction is small. The resonant condition $\omega_0\simeq\Omega_{c,v}$ requires to take into account Landau levels with $n\geq1$. It can be seen that all Landau levels could be taken into account by rewriting the BS equation in the form $G^{-1}G^{-1}\Psi=I\Psi$,[17,18] but in this case there is no dimensional reduction. In what follows we assume that (i) the resonant condition does not hold, and therefore, the contributions to the Green's functions from the excited Landau levels are negligible, and (ii) we consider the case of a strong magnetic field (exciton binding energy much smaller than the exciton cyclotron energy). In this regime, one can apply the LLL approximation, according to which one can ignore transitions between Landau levels and consider only the states on the lowest Landau level, i.e., we keep only the $n=0$ term in the last expressions,

$$\widetilde{G}_c(\mathbf{k};k_z;\hbar\omega)\approx\frac{2\exp(-l^2\mathbf{k}^2)}{\hbar\omega-[E_g+\hbar^2k_z^2/2m_c+\hbar\Omega_c/2]+\imath0^+},\widetilde{G}_v(\mathbf{k};k_z;\hbar\omega)\approx\frac{2\exp(-l^2\mathbf{k}^2)}{\hbar\omega+[\hbar^2k_z^2/2m_v+\hbar\Omega_v/2)}-\imath0^+. \tag{7}$$

The solution of the BS equation in the LLL approximation can be written in the following form:

$$\tilde{\chi}_{\mathbf{Q},Q_z}(\mathbf{k};k_z;\omega)=\exp\left[-l^2\left(\mathbf{k}+\frac{\gamma}{2}\mathbf{Q}\right)^2-\imath\mathbf{Q}_0\cdot\mathbf{k}l^2\right]\varphi_{Q_z}(k_z;\omega). \tag{8}$$

Thus, the LLL approximation reduces the problem from 3+1 dimensions to 1+1 dimensions, and therefore, the functions $\varphi_{Q_z}(k_z;\omega)$ and the energy $E(\mathbf{Q},Q_z)$ can be obtained from the following equation:

$$\varphi_{Q_z}(k_z;\omega)=\int\frac{dp_z}{2\pi}\frac{d\Omega}{2\pi}\frac{d\omega}{2\pi}I_{\mathbf{Q}}(p_z-k_z;\omega-\Omega)\left\{\frac{1}{\hbar\omega+\alpha_cE-\left[E_g+\dfrac{\hbar^2}{2m_c}(k_z+\alpha_cQ_z)^2+\dfrac{\hbar\Omega_c}{2}\right]+\imath0^+}\right.$$
$$\left.+\frac{1}{\hbar\omega-\alpha_vE+\dfrac{\hbar^2}{2m_v}(k_z-\alpha_vQ_z)^2+\dfrac{\hbar\Omega_v}{2}-\imath0^+}\right\}\varphi_{Q_z}(p_z;\Omega).$$

In the LLL approximation, the exciton dispersion is determined by the term

$$I_{\mathbf{Q}}(q_z;\omega)=\frac{4\pi e^2}{\varepsilon_\infty}\int\frac{d^2\mathbf{q}}{(2\pi)^2}\int d^2\mathbf{r}\frac{\psi_{00}^2(\mathbf{r})e^{\imath\mathbf{q}\cdot(\mathbf{r}+l^2\mathbf{Q}_0)}}{q^2+q_z^2}\left[1-\frac{\omega_0}{2}\frac{\varepsilon_\infty}{\varepsilon^*}\left(\frac{1}{\omega_0-\omega-\imath0^+}+\frac{1}{\omega_0+\omega-\imath0^+}\right)\right]. \tag{9}$$

The solution of Eq. (8) can be chosen in the following form:

$$\varphi_{Q_z}(k_z,\omega)=\frac{\phi(k_z)}{\left\{\hbar\omega+\alpha_cE-\left[E_g+\dfrac{\hbar^2}{2m_c}(k_z+\alpha_cQ_z)^2+\dfrac{\hbar\Omega_c}{2}\right]+\imath0^+\right\}\left\{\hbar\omega-\alpha_vE+\left[\dfrac{\hbar^2}{2m_v}(k_z-\alpha_vQ_z)^2+\dfrac{\hbar\Omega_v}{2}\right]-\imath0^+\right\}}. \tag{10}$$

Integrating both sides of Eq. (9) over ω, we find the following equation for the exciton wave function $\Phi_{Q_z}(k_z)$ and exciton energy $E(\mathbf{Q},Q_z)=E_g+\frac{1}{2}\hbar\Omega+\hbar^2Q_z^2/2M-E_b(\mathbf{Q},Q_z)$:

$$-E_b(\mathbf{Q},Q_z)\Phi_{Q_z}(k_z)=\frac{\hbar^2k_z^2}{2\mu}\Phi_{Q_z}(k_z)-\frac{4\pi e^2}{\varepsilon_\infty}\int\frac{dq_z}{2\pi}\frac{d^2\mathbf{q}}{(2\pi)^2}\int d^2\mathbf{r}\frac{\psi_{00}^2(\mathbf{r})e^{\imath\mathbf{q}\cdot(\mathbf{r}+l^2\mathbf{Q}_0)}}{q^2+(k_z-q_z)^2}\Phi_{Q_z}(q_z)$$
$$+\frac{2\pi e^2}{\varepsilon^*}\int\frac{dq_z}{2\pi}\frac{d^2\mathbf{q}}{(2\pi)^2}\int d^2\mathbf{r}\frac{\psi_{00}^2(\mathbf{r})e^{\imath\mathbf{q}\cdot(\mathbf{r}+l^2\mathbf{Q}_0)}}{q^2+(k_z-q_z)^2}\Phi_{Q_z}(q_z)$$
$$\times\left[\frac{\hbar\omega_0}{\hbar\omega_0+E_b(\mathbf{Q},Q_z)+\Delta_{Q_z}(q_z,k_z)}+\frac{\hbar\omega_0}{\hbar\omega_0+E_b(\mathbf{Q},Q_z)+\Delta_{Q_z}(k_z,q_z)}\right],$$

where

$$\Delta_{Q_z}(k_z,q_z)=\frac{\hbar^2k_z^2}{2m_c}+\frac{\hbar^2q_z^2}{2m_v}+\frac{\hbar^2Q_z(k_z-q_z)}{M}.$$

In position representation the last equation assumes the form

$$-E_b(\mathbf{Q},Q_z)\Phi_{Q_z}(z)=-\frac{\hbar^2}{2\mu}\frac{d^2\Phi_{Q_z}(z)}{dz^2}-V_{\mathbf{Q}}(z)\Phi_{Q_z}(z)+\int_{-\infty}^{\infty}dz'U_{\mathbf{Q},Q_z}[z,z';E_b(\mathbf{Q},Q_z)]\Phi_{Q_z}(z'), \tag{11}$$

where in the LLL approximation

$$V_{\mathbf{Q}}(z) = \frac{e^2}{\varepsilon_\infty}\int d^2\mathbf{r}\frac{\psi_{00}^2(\mathbf{r})}{\sqrt{(\mathbf{r}+l^2\mathbf{Q}_0)^2+z^2}}. \tag{12}$$

Here, $\psi_{00}(\mathbf{r})=\exp(-r^2/4l^2)/\sqrt{2\pi l^2}$ is the LLL wave function. Thus, we obtain a Schrödinger type of equation but with an extra nonlocal potential,

$$U_{\mathbf{Q},Q_z}[z,z';E_b(\mathbf{Q},Q_z)] = \frac{2\pi e^2}{\varepsilon^*}\int\frac{dk_z}{2\pi}\frac{dq_z}{2\pi}\frac{d^2\mathbf{q}}{(2\pi)^2}\int d^2\mathbf{r}\frac{\psi_{00}^2(\mathbf{r})\exp\{\imath[\mathbf{q}\cdot(\mathbf{r}+l^2\mathbf{Q}_0)+k_z z-q_z z']\}}{q^2+(k_z-q_z)^2}$$
$$\times\left[\frac{\hbar\omega_0}{\hbar\omega_0+E_b(\mathbf{Q},Q_z)+\Delta_{Q_z}(q_z,k_z)}+\frac{\hbar\omega_0}{\hbar\omega_0+E_b(\mathbf{Q},Q_z)+\Delta_{Q_z}(k_z,q_z)}\right],$$

which represents the effect of the exciton-LO-phonon interaction. It is worth mentioning that because of the interaction with the LO phonons, the in-plane motion and the motion along the z direction are not independent.

For $\mathbf{Q}=0$ and $Q_z=0$, after integrations over $\mathbf{r}$ and $\mathbf{q}$, we find the following equation for the magnetoexciton binding energy $\epsilon=E_b/\hbar\Omega$ and the corresponding wave function $\Phi(z)$ (z is in units of l):

$$0 = \left[-\frac{1}{2}\frac{d^2}{dz^2}-V_0(z)+\epsilon\right]\Phi(z)+\int_{-\infty}^{\infty}dz'U_{0,0}(z,z';\epsilon)\Phi(z'). \tag{13}$$

Here, $V_0(z)=\sqrt{\frac{\pi}{2}}(l/a_B)Erfc(\frac{|z|}{\sqrt{2}l})\exp(\frac{z^2}{2l^2})$ and the nonlocal potential (in units of $\hbar\Omega$) is

$$U_{0,0}(z,z';\epsilon) = \frac{1}{2}(l/a_B)\beta\left(1-\frac{\varepsilon_\infty}{\varepsilon_0}\right)\int\frac{dk_z}{2\pi}\frac{dq_z}{2\pi}\exp\{\imath[k_z z+q_z(z-z')]\}\exp\left(\frac{k_z^2}{2}\right)\Gamma\left(0,\frac{k_z^2}{2}\right)$$
$$\times\left[\left\{\beta+\epsilon+\frac{1}{2}[\alpha_c(k_z+q_z)^2+\alpha_v q_z^2]\right\}^{-1}+\left\{\beta+\epsilon+\frac{1}{2}[\alpha_v(k_z+q_z)^2+\alpha_c q_z^2]\right\}^{-1}\right],$$

where $\beta=\omega_0/\Omega$, and $a_B=\varepsilon_\infty\hbar^2/\mu e$ is the exciton Bohr energy. $Erfc(x)$ and $\Gamma(a,x)$ are the complementary error function and the incomplete gamma function, respectively.

The last term in Eq. (13) is nonlocal in space and depends on the binding energy and the electron and the hole bare masses. The dependence on the bare masses is due to the $3+1\rightarrow 1+1$ reduction, so one can expect that in a pure 2D case the nonlocal potential should be the mass-independent one.

Without the nonlocal term, Eq. (13) has been studied decades ago by many authors.[8] More recently, it was found (see, e.g., Ref. 19 and references therein) that the eigenvalues can be separated into two distinct classes: the states having no node and the states having node (or notes) in their eigenfunctions. The states having no node in their wave functions are tightly bound while the states having nodes in their wave functions are weakly bound. A complete numerical evaluation of Eq. (13) for arbitrary value of the magnetic field is a complicated problem beyond the main goal of this paper.

III. QUANTUM-WELL MAGNETOEXCITONS

Our approach can be applied not only to bulk crystals but also to the quasi-two-dimensional systems as well. In quantum wells, however, besides the bulklike phonon modes one has to take into account the presence of slab modes,[20] interface modes,[21] and half-space modes.[22] In quasi-two-dimensional systems we still have a dimensional reduction $2+1\rightarrow 0+1$, but each of the above modes will create extra poles in the inverse dielectric function $\varepsilon^{-1}(\mathbf{q},\omega)$. The problem becomes too complicated and cannot be solved analytically.

In what follows we calculate the magnetoexciton dispersion in a quantum well, taking into account only the interaction with Fröhlich's bulk LO phonons. In other words, we shall take into account the effect of size quantization in a quantum well on the electron (hole) spectrum, whereas the phonon spectrum stays the same as in a homogeneous medium, as if the entire space was filled with the quantum-well material. Strictly speaking, this approach can provide only an approximate solution of the quasi-two-dimensional problem. But, we expect that our results can be used in the case of narrow quantum wells because they are exact in a pure 2D case.

In a single quantum well (SQW) and coupled quantum wells (CQWs) the Fourier transform of the exciton wave function satisfies the following BS equation:

$$\tilde{\chi}_{\mathbf{Q}}\left(\mathbf{k}-\frac{\gamma}{2}\mathbf{Q};\omega\right)=\int\frac{d^2\mathbf{q}}{(2\pi)^2}\frac{d^2\mathbf{p}}{(2\pi)^2}d^2\mathbf{R}\int_{-\infty}^{\infty}\frac{d\Omega}{2\pi}\exp[-\imath(\mathbf{q}+\mathbf{Q})\cdot\mathbf{R}]\tilde{G}_c\left[\frac{1}{2}\mathbf{q}+\mathbf{k}-\frac{e}{\hbar c}\mathbf{A}(\mathbf{R});\hbar\omega+\alpha_c E\right]$$
$$\times\tilde{G}_v\left[-\frac{1}{2}\mathbf{q}+\mathbf{k}-\frac{e}{\hbar c}\mathbf{A}(\mathbf{R});\hbar\omega-\alpha_v E\right]V\left\{\mathbf{p}-\left[\mathbf{k}-\frac{2e}{\hbar c}\mathbf{A}(\mathbf{R})\right];\omega-\Omega\right\}\tilde{\chi}_{\mathbf{Q}}\left(\mathbf{p}-\frac{\gamma}{2}\mathbf{Q};\Omega\right),\tag{14}$$

where $V(\mathbf{k};\omega)=-[2\pi e^2 f(|\mathbf{k}|)/|\mathbf{k}|]\varepsilon^{-1}(\omega)$, where $f(\mathbf{k})$ is the structure factor

$$f(|\mathbf{q}|)=f(q)=\int_{-\infty}^{+\infty}dz_c\int_{-\infty}^{+\infty}dz_v\,\exp\{-q|z_c-z_v|]\}\varphi_{0c}^2(z_c)\phi_{0v}^2(z_v).\tag{15}$$

In our calculations, we take into account only the first electron E_{0c} and hole E_{0v} confinement levels with wave functions $\varphi_{0c}(z_c)$ and $\phi_{0v}(z_v)$, respectively.

In the LLL approximation the exact fermion Green's functions $G_{c,v}$ are replaced by the corresponding propagator of the free fermions $G_{c,v}^{(0)}$,

$$\tilde{G}_c(\mathbf{k};\hbar\omega)\approx\frac{2\exp(-l^2\mathbf{k}^2)}{\hbar\omega-[E_g+E_{0c}+\hbar\Omega_c/2]+\imath 0^+},$$

$$\tilde{G}_v(\mathbf{k};\hbar\omega)\approx\frac{2\exp(-l^2\mathbf{k}^2)}{\hbar\omega+E_{0v}+\hbar\Omega_v/2-\imath 0^+}.\tag{16}$$

The solution of the BS equation in the LLL approximation can be written in the following form:

$$\tilde{\chi}_{\mathbf{Q}}(\mathbf{k};\omega)=\exp\left[-l^2\left(\mathbf{k}+\frac{\gamma}{2}\mathbf{Q}\right)^2-\imath\mathbf{Q}_0\cdot\mathbf{k}l^2\right]\varphi_E(\omega).\tag{17}$$

Thus, the LLL approximation reduces the problem from 2+1 dimensions to one-dimensional problem for obtaining function $\varphi(\omega)$ and energy $E(\mathbf{Q})$ from the following equation:

$$\varphi_E(\omega)=-\frac{1}{\left[\hbar\omega+\alpha_c E-E_g-E_{0c}-\frac{\hbar\Omega_c}{2}+\imath 0^+\right]\left[\hbar\omega-\alpha_v E+E_{0v}+\frac{\hbar\Omega_v}{2}-\imath 0^+\right]}$$
$$\times\left[I(|\mathbf{Q}|,\varepsilon_\infty)\int_{-\infty}^{\infty}\frac{d\Omega}{2\pi}\varphi_E(\Omega)-I(|\mathbf{Q}|,\varepsilon^*)\int_{-\infty}^{\infty}\frac{d\Omega}{2\pi}\varphi_E(\Omega)\frac{\omega_0}{2}\left(\frac{1}{\omega_0-\omega+\Omega-\imath 0^+}+\frac{1}{\omega_0+\omega-\Omega-\imath 0^+}\right)\right].\tag{18}$$

The exciton dispersion is determined by the term

$$I(\mathbf{Q},\varepsilon)=\frac{2\pi e^2}{\varepsilon}\int d^2\mathbf{r}\frac{d^2\mathbf{q}}{(2\pi)^2}\psi_{00}^2(\mathbf{r})\frac{f(|\mathbf{q}|)e^{\imath\mathbf{q}\cdot(\mathbf{r}+l^2\mathbf{Q}_0)}}{|\mathbf{q}|}.\tag{19}$$

The solution of Eq. (18) can be chosen in the following form:

$$\varphi_E(\omega)=\frac{1}{\left[\hbar\omega+\alpha_c E-E_g-E_{0c}-\frac{\hbar\Omega_c}{2}+\imath 0^+\right]\left[\hbar\omega-\alpha_v E+E_{0v}+\frac{\hbar\Omega_v}{2}-\imath 0^+\right]}.\tag{20}$$

Thus, by integrating both sides of Eq. (19) over ω, we find the following equation for the exciton dispersion $E(|\mathbf{Q}|)=E_g+E_{0c}+E_{0v}+\hbar\Omega/2-E_b(|\mathbf{Q}|)$:

$$E_b(|\mathbf{Q}|)=I(|\mathbf{Q}|,\varepsilon_\infty)-I(|\mathbf{Q}|,\varepsilon^*)\frac{\hbar\omega_0}{\hbar\omega_0+E_b(|\mathbf{Q}|)}.\tag{21}$$

Solving for $E_b(|\mathbf{Q}|)$, we obtain

$$E_b(|\mathbf{Q}|) = \frac{1}{2}\Bigg[I(|\mathbf{Q}|,\varepsilon_\infty) - \hbar\omega_0 + \sqrt{(I(|\mathbf{Q}|,\varepsilon_\infty) - \hbar\omega_0)^2 + 4\frac{\varepsilon_\infty}{\varepsilon_0} I(|\mathbf{Q}|,\varepsilon_\infty)\hbar\omega_0}\Bigg]. \tag{22}$$

When $\mathbf{Q}=0$ the exciton binding energy $E_b=E_b(\mathbf{Q}=0)$ is

$$E_b = \frac{I_\infty}{2}\left[1 - \frac{\hbar\omega_0}{I_\infty} + \sqrt{\left(1-\frac{\hbar\omega_0}{I_\infty}\right)^2 + 4\frac{\varepsilon_\infty}{\varepsilon_0}\frac{\hbar\omega_0}{I_\infty}}\right], \tag{23}$$

where $I_\infty = I(|\mathbf{Q}|=0,\varepsilon_\infty)$.

In a pure 2D case the exciton dispersion is determined by the terms $I_{2D}(\mathbf{Q},\varepsilon_\infty)$ and $I_{2D}(\mathbf{Q},\varepsilon^*)$, where

$$I_{2D}(\mathbf{Q},x) = \frac{2\pi e^2}{x}\int d^2\mathbf{r}\frac{d^2\mathbf{q}}{(2\pi)^2}\psi_{00}^2(\mathbf{r})\frac{e^{i\mathbf{q}\cdot(\mathbf{r}+l^2\mathbf{Q}_0)}}{|\mathbf{q}|}.$$

The exciton dispersion in 2D $E(|\mathbf{Q}|)=E_g+\hbar\Omega/2-E_b(|\mathbf{Q}|)$ becomes

$$E_b(|\mathbf{Q}|) = \frac{1}{2}\Bigg[I_{2D}(|\mathbf{Q}|,\varepsilon_\infty) - \hbar\omega_0 + \sqrt{(I_{2D}(|\mathbf{Q}|,\varepsilon_\infty) - \hbar\omega_0)^2 + 4\frac{\varepsilon_\infty}{\varepsilon_0} I_{2D}(|\mathbf{Q}|,\varepsilon_\infty)\hbar\omega_0}\Bigg]. \tag{24}$$

Let us compare our results with the binding energy $E_{2D}=\sqrt{\pi/2}(e^2/\epsilon l)$ and the in-plane mass $M_{2D}=2^{3/2}\epsilon/\sqrt{\pi e^2 l}$ of 2D magnetoexcitons in nonpolar materials.[23] When $\mathbf{Q}=0$ the magnetoexciton binding energy (22) assumes a similar form, $E_b=\sqrt{\frac{\pi}{2}}e^2/\epsilon(B)l$, but with an effective dielectric constants,

$$\epsilon(B) = \frac{2\varepsilon_\infty}{1 - \frac{\hbar\omega_0}{E_{2D}} + \sqrt{\left(1-\frac{\hbar\omega_0}{E_{2D}}\right)^2 + 4\frac{\varepsilon_\infty}{\varepsilon_0}\frac{\hbar\omega_0}{E_{2D}}}},. \tag{25}$$

In the high-field ($B\to\infty$) limit the effective dielectric constant (25) approaches the high-frequency dielectric constant. For strong magnetic fields when the inequality $E_{2D}>\hbar\omega_0$ holds, the effective dielectric constant is lower than the static dielectric constant. Only for magnetic fields such that $\hbar\omega_0 \gg E_{2D}$, the interaction between the electrons and holes is screened by the static dielectric constant.

For small wave vectors, the magnetoexciton dispersion is parabolic and is characterized by an effective magnetoexciton mass which can be calculated from the dispersion Eq. (22),

$$M_{2D} = \frac{2^{5/2}\varepsilon_\infty}{\sqrt{\pi}e^2 l}\left[1 + \frac{1 + \left(2\frac{\varepsilon_\infty}{\varepsilon_0} - 1\right)\frac{\hbar\omega_0}{E_{2D}}}{\sqrt{\left(1-\frac{\hbar\omega_0}{E_{2D}}\right)^2 + 4\frac{\varepsilon_\infty}{\varepsilon_0}\frac{\hbar\omega_0}{E_{2D}}}}\right]^{-1}.$$

Table I gives the result of our numerical calculation of the effective dielectric constant (25) for various strong magnetic fields B. For GaAs the input parameters were measured at $T=75.6$ K, so our predicted result for the effective dielectric constant at $B=4$ T is in a good agreement with the value used in Ref. 11.

In Ref. 18, the magnetoexciton binding energies for magnetic fields $4<B<20$ T have been calculated by a variational method without taking into account the interaction with LO phonons. There are two major differences between the present work and the previous one. The first difference is that in Ref. 18 the interaction with the LO phonons was neglected. Instead, an effective dielectric constant ϵ_∞ for GaAs quantum wells has been used. The second difference is that in Ref. 18 all Landau levels have been taken into account. Nevertheless the previous and the present calculations are not the same; we can see that when the magnetic field decreases from $B=20$ T to $B=4$ T, the difference between the calculated binding energies in Ref. 18 and the experimental values also decreases. This effect can be attributed to the fact that the error due to the above-chosen dielectric constant decreases because the difference between $\epsilon(B)$ and $\epsilon_\infty=12.35$ also decreases when magnetic field changes from 20 to 4 T. It should be pointed out that variational calculations of the heavy-hole exciton ground-state energies in GaAs quantum wells for various well widths L and strong magnetic fields (see Table II in Ref. 18) can be improved by taking into account the interaction with the LO phonons. For example, in the case of $L=4.03$ nm single quantum well the average Coulomb energies $V_C(\beta)$ calculated by the variational method for $B=20$, 18, and 16 T are as follows: 23.7, 22.9, and 21.8 meV, respectively. When the interaction with LO phonons is taken into account, the corresponding average Coulomb energies calculated by means of Eq. (23) with $\epsilon_\infty=10.89$, $\epsilon_0=12.74$, and $\hbar\omega_0=36.8$ meV are 24.8, 24.0, and 22.8 meV, respectively. Thus, the position of the exciton ground state moves about 1 meV closer to the experimental values.

IV. SUMMARY

Generally speaking, the interaction with LO phonons can strongly influence the optical and transport properties of weak polar crystals, such as III-V semiconductors. In the present paper we have formulated a different approach to the problem of magnetoexcitons interacting with LO phonons which in 2D case provides analytical results for the binding energy and the exciton mass in terms of three parameters: ε_∞, $\frac{\varepsilon_\infty}{\varepsilon_0}$, and $\frac{\hbar\omega_0}{E_{2D}}$. The reason to obtain analytical results lies in the fact that the dynamics of the lowest Landau level is essentially $D-2$ dimensional.

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